Solutions

Math 220

HW # 2

September 26, 2018

Exercise 1. *Determine which of the following are statements. If not, explain why not. If so, determine the truth value of the statement.*

(a) *Calvin Coolidge was the greatest American President.*

(b) *The square root of a rational number is always a rational number.*

(c) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

(d) *Mixing yellow and red paint will give you orange paint.*

(e) *Life is like a box of chocolates.*

(f) *When will the Red Sox win the World Series?*

(g) *This sentence is false.*

(h) *A group of owls is called a parliament.*

(i) $\frac{n(n+1)}{2}$.

(j) *Every former President of the United States is buried in the United States.*

(k) *Everyone has a cat.*

Solution.

(a) This is not a statement, since it is an opinion.

(b) This is a false statement. (2 is rational, but $\sqrt{2}$ is irrational.)

(c) This is a true statement. (We'll prove this later.)

(d) This is a true statement.

(e) This is not a statement, but an opinion.

(f) This is not a statement, it is a question.

(g) This is not a statement because the truth value is indeterminant (it's a paradox)!

(h) This is a true statement.

(i) This is not a statement because no claim is being made.

(j) This is a false statement as living former presidents are not buried.

(k) This is a false statement.

□

Exercise 2. *Write the negation of each of the following statements, without just putting the phrase "It is not the case that..." in front of the given phrase.*

(a) *Pi is a positive real number.*

(b) *Georgia is the eleventh largest state.*

(c) *Flatland State University has no major in paleontology.*

(*d*) $3 + 1 < 4$.
(*e*) 3 *is a factor of* 7.
(*f*) $1 + 2 + 3 = \frac{3(3+1)}{2}$.
(*g*) *Sam is an orange belt and Kate is a red belt.*
(*h*) *The train is late or my watch is fast.*

Solution.

(*a*) Pi is not a positive real number.
(*b*) Georgia is not the eleventh largest state.
(*c*) Flatland State University has a major in paleontology.
(*d*) $3 + 1 \geq 4$.
(*e*) 3 is not a factor of 7.
(*f*) $1 + 2 + 3 \neq \frac{3(3+1)}{2}$.
(*g*) Sam is not an orange belt or Kate is not a red belt.
(*h*) The train is not late and my watch is not fast.

□

Exercise 3. *Write a truth table for the statement*

$$\neg p \wedge q.$$

Solution.

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

□

Exercise 4. *Write a truth table for the statement*

$$p \wedge (\neg q \vee r)$$

Solution.

p	q	r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

□

Exercise 5. *Determine whether the given pair is logically equivalent. Justify your answer using truth tables and include a few words of explanation.*

$$p \vee (p \wedge q) \text{ and } p.$$

Solution.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Comparing the columns for both p and $p \vee (p \wedge q)$, we see that they have the same truth values, and therefore they are logically equivalent. □

Exercise 6. *Determine whether the given pair is logically equivalent. Justify your answer using truth tables and include a few words of explanation.*

$$(p \vee q) \vee (p \wedge r) \text{ and } (p \vee q) \wedge r.$$

Solution.

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Since $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ have different columns in the truth table, they are not logically equivalent. □

Exercise 7. *Determine whether the following statement is a tautology or a contradiction. Justify your answer using truth tables and include a few words of explanation.*

$$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q)).$$

Solution.

p	q	$\neg q$	$p \wedge \neg q$	$\neg p$	$\neg p \vee (p \wedge \neg q)$	$p \wedge q$	$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$
T	T	F	F	F	F	T	T
T	F	T	T	F	T	F	T
F	T	F	F	T	T	F	T
F	F	T	F	T	T	F	T

The the expression $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ has T for a truth value in every row, it is a tautology. □

Exercise 8. *Determine whether the following statement is a tautology or a contradiction. Justify your answer using truth tables and include a few words of explanation.*

$$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q.$$

Solution.

p	q	r	$\neg p$	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$\neg q$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
T	T	T	F	F	T	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	F	F	F	T	F
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	F	F
F	T	F	T	T	F	F	F	F
F	F	T	T	F	F	F	T	F
F	F	F	T	F	F	F	T	F

Since every entry in the truth table for $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$ is F, it is a contradiction. □

Exercise 9. *Below, a logical equivalence is derived using Theorem 1.1.1 from Epp's book. Supply a reason for each step (which part of the theorem is used).*

$$\begin{aligned}(p \wedge \neg q) \vee (p \wedge q) &\equiv p \wedge (\neg q \vee q) &&\text{by } \underline{(a)}\\ &\equiv p \wedge (q \vee \neg q) &&\text{by } \underline{(b)}\\ &\equiv p \wedge \mathbf{t} &&\text{by } \underline{(c)}\\ &\equiv p &&\text{by } \underline{(d)}\end{aligned}$$

Therefore, $(p \wedge \neg q) \vee (p \wedge q) \equiv p$.

Solution.

(a) distributivity laws
(b) commutativity laws
(c) negation laws
(d) identity laws

□

Exercise 10. *Use Theorem 1.1.1 from Epp to verify the logical equivalence. Give a reason for each step (which part of the theorem is used).*

$$(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p.$$

Solution.

$$\begin{aligned}
(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) &\equiv (p \wedge (p \wedge \neg q)) \vee (p \wedge q) && \text{by DeMorgan's Laws} \\
&\equiv ((p \wedge p) \wedge \neg q) \vee (p \wedge q) && \text{by Associativity of } \wedge \\
&\equiv (p \wedge \neg q) \vee (p \wedge q) && \text{by Idempotent Laws} \\
&\equiv p \wedge (\neg q \vee q) && \text{by Distributive Laws} \\
&\equiv p \wedge \mathbf{t} && \text{by Negation Laws} \\
&\equiv p && \text{by Identity Laws}
\end{aligned}$$

Therefore, $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$. □